Fast and secure DH implementation

Chitchanok Chuengsatiansup

Technische Universiteit Eindhoven

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Diffie–Hellman key exchange

a b

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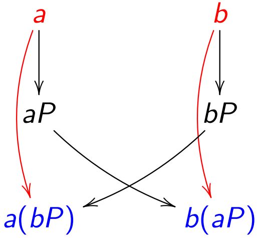
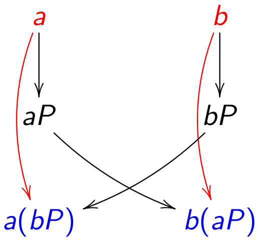


Diffie–Hellman key exchange



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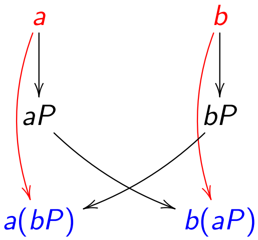
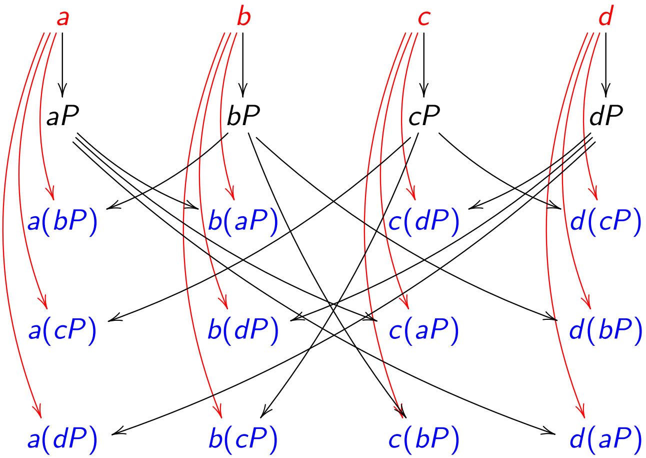
Diffie–Hellman key exchange



Diffie–Hellman key exchange



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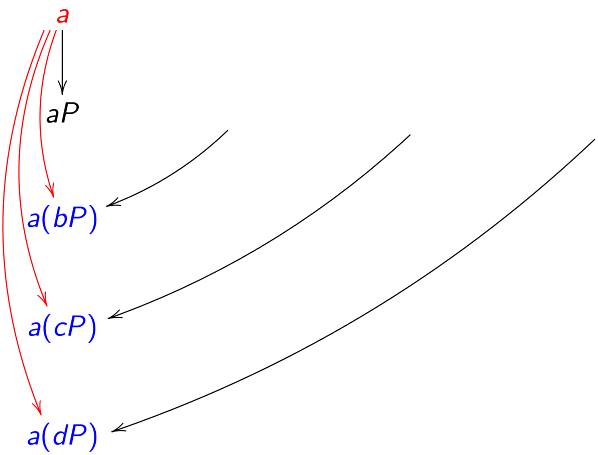
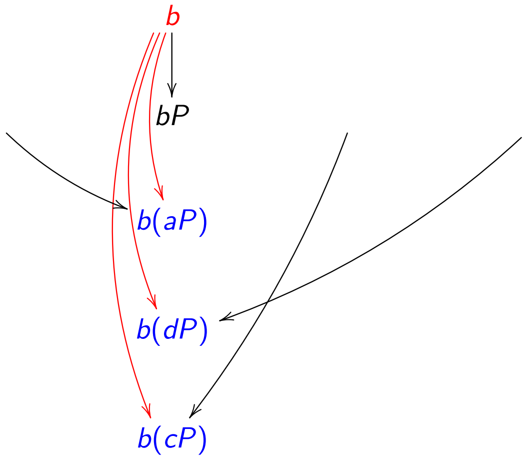


main DH challenge: make variable-base scalar mult as fast as possible



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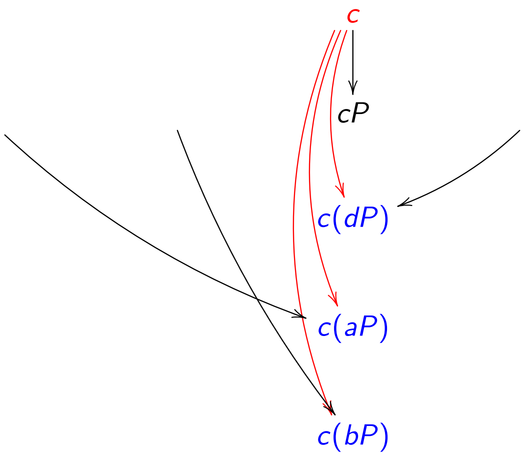
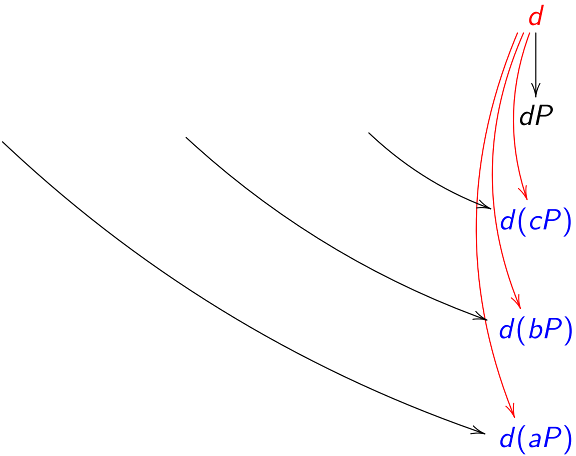
Diffie–Hellman key exchange



main DH challenge: make variable-base scalar mult as fast as possible

Diffie–Hellman key exchange



main DH challenge: make variable-base scalar mult as fast as possible

Scalar multiplication

- Given scalar n and point P
- Compute $nP = \underbrace{P + P + \ldots + P}_{n}$

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- Given scalar n and point P
- Compute $nP = \underbrace{P + P + \ldots + P}_{n}$
- Algorithms:
  - Double-and-add
  - Ladder

Double-and-add

Input: $P, n = (n_{i-1}, \ldots, n_0)_2$
Output: $R = nP$

$R \leftarrow 1P$

for $c = n_{i-2}$ **to** n_0
  $R \leftarrow 2R$
  if $c = 1$ **then**
    $R \leftarrow R + P$

Return R

Example: Compute $9P$

$9 = 1001_2$

$R = \quad 1P$

$n_2 = 0 : \quad 2P$

$n_1 = 0 : \quad 4P$

$n_0 = 1 : \quad 8P + P = 9P$

Windowing method

Input: $P, n = (n_{i-1}, \ldots, n_0)_2$
Output: $R = nP$

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convert n to radix 2^ω:
$n = (c_j, \ldots, c_0)_{2^\omega}$
$R \leftarrow c_j P$

for $k = j - 1$ **down to** 0
$R \leftarrow 2^\omega R + c_k P$

Return R

Note: ω = window width

Example: Compute $2345P$

$2345 = \underline{10}\ \underline{01001}\ \underline{01001}\ _2$

$= 299_{32}$

$R = 10_2 = 2P$

$01001_2 = 9$:

$2^5(2P) + 9P = 73P$

$01001_2 = 9$:

$2^5(72P) + 9P = 2345P$

Ladder

Input: $P, n = (n_{i-1}, \ldots, n_0)_2$
Output: $R_0 = nP$

$R_0 \leftarrow 0P; \quad R_1 \leftarrow 1P$
if $n_i = 0$ **then**
$\quad R_1 \leftarrow R_0 + R_1; \quad R_0 \leftarrow 2R_0$
else
$\quad R_0 \leftarrow R_0 + R_1; \quad R_1 \leftarrow 2R_1$

Return R_0

Example: Compute $9P$

$9 = 1001_2$

$(R_0, R_1) = (0P, 1P)$

$n_3 = 1: \quad (1P, 2P)$

$n_2 = 0: \quad (2P, 3P)$

$n_1 = 0: \quad (4P, 5P)$

$n_0 = 1: \quad (9P, 10P)$

Side-channel attacks

- Try to discover secret information via physical measurements such as
  - electromagnetic radiation
  - power consumption
  - run time
  - noise

Side-channel attacks

- Try to discover secret information via physical measurements such as
  - electromagnetic radiation
  - power consumption
  - run time
  - noise
- Examples of side-channel attacks are:
  - timing attack
  - power attack
  - fault attack

Secure the implementation

- Prevent software side-channel attacks:
  - constant-time
  - no input-dependent branch
  - no input-dependent array index

Secure the implementation

- Prevent software side-channel attacks:
  - constant-time
  - no input-dependent branch
  - no input-dependent array index
- Constant-time table-lookup:
  - read entire table
  - select via arithmetic
    if c is 1, select tbl[i]
    if c is 0, ignore tbl[i]

$$t = (t \cdot (1-c)) + (\text{tbl[i]} \cdot (c))$$

$$t = (t \ \texttt{and} \ (c-1)) \ \texttt{xor} \ (\text{tbl[i]} \ \texttt{and} \ (-c))$$

Vectorization speedups

without vector

a

$+$

b

$=$

$a + b$

Vectorization speedups

without vector

a
$+$
b
$=$
$a + b$

with vector

a_0	a_1	a_2	a_3
$+$	$+$	$+$	$+$
b_0	b_1	b_2	b_3
$=$	$=$	$=$	$=$
$a_0 + b_0$	$a_1 + b_1$	$a_2 + b_2$	$a_3 + b_3$

Vectorization speedups

without vector	with vector			
a	a_0	a_1	a_2	a_3
$+$	$+$	$+$	$+$	$+$
b	b_0	b_1	b_2	b_3
$=$	$=$	$=$	$=$	$=$
$a + b$	$a_0 + b_0$	$a_1 + b_1$	$a_2 + b_2$	$a_3 + b_3$

- **single** instruction performing n **independent** operations on **aligned** inputs

Based on

Kummer strikes back: new DH speed records
Joint work with Daniel J. Bernstein, Tanja Lange & Peter Schwabe

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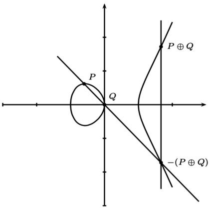
Security level is $\approx \sqrt{p^g}$ where p is the field size and g is the genus.
We work on Kummer surface of genus-2 hyperelliptic curve over $\mathbf{F}_p$ where $p = 2^{127} - 1$, i.e., $\approx 2^{127}$ security.
Scalar multiplication is computed using ladder.

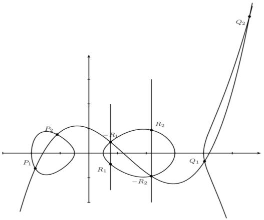
ECC



$$y^2 = x^3 + ax + b$$

$\longrightarrow$ x-line represented as $(X : Z)$

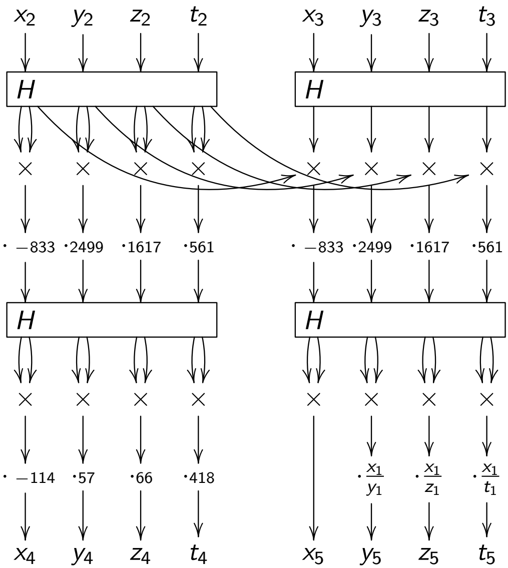
HECC



$$v^2 = u^5 + f_4 u^4 + f_3 u^3 + f_2 u^2 + f_1 u^1 + f_0$$

$\longrightarrow$ Kummer surface represented as $(X : Y : Z : T)$

Squared Kummer surface ladder



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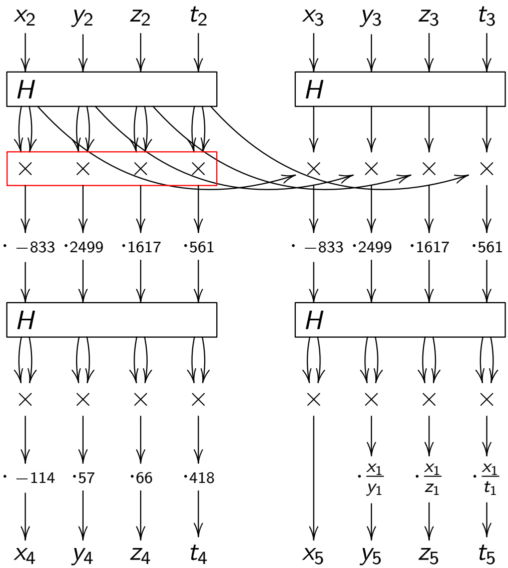

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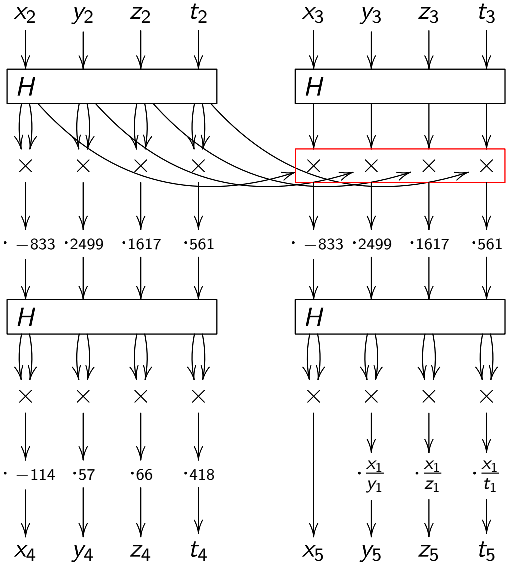

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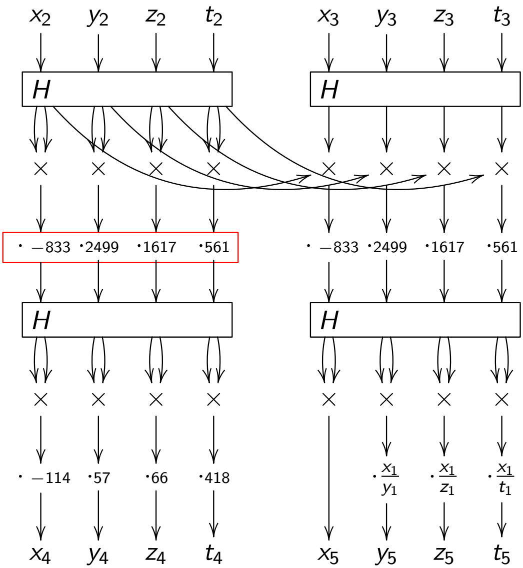

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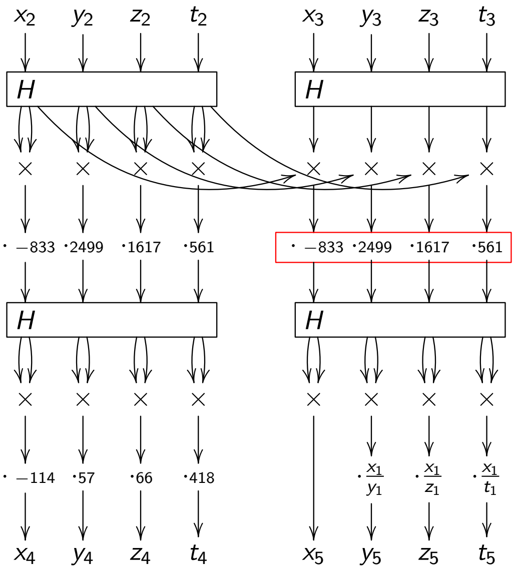

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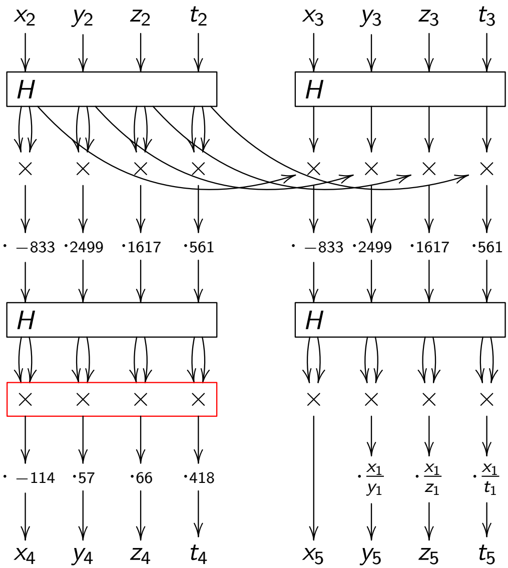

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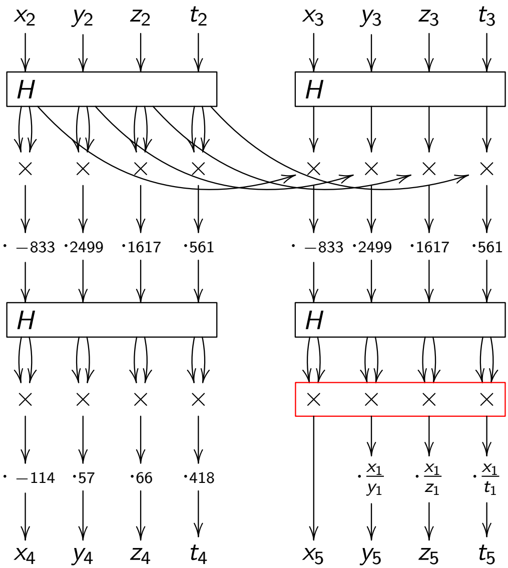

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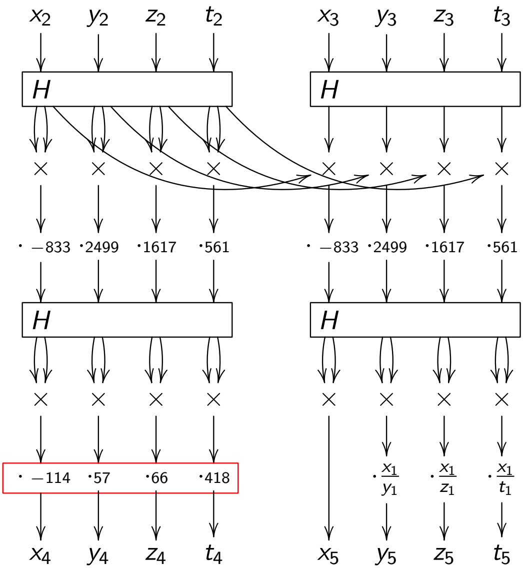

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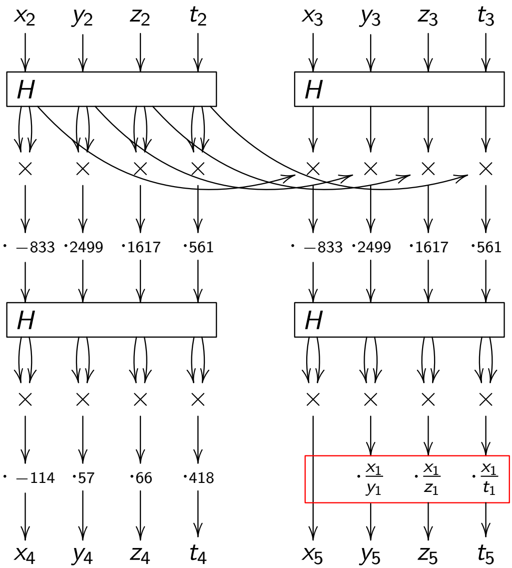


Performance comparison (1/2)

Arch	Cycles	g	Field	Source of software
TI Sitara	497389	1	$2^{255}-19$	BeSc CHES 2012
TI Sitara	305395	2	$2^{127}-1$	**BeChLaSc** Asiacrypt 2014
i.MX515	460200	1	$2^{255}-19$	BeSc CHES 2012
i.MX515	273349	2	$2^{127}-1$	**BeChLaSc** Asiacrypt 2014

picture credit:

http://www.pngall.com/wp-content/uploads/2016/03/Smartphone-PNG-Pic.png

Performance comparison (2/2)

Arch	Cycles	g	Field	Source of software	
Sandy	122716	2	$2^{127}-1$	BoCoHiLa	Eurocrypt 2013
Sandy	119904	1	2^{254}	OlLóArRo	CHES 2013
Sandy	96000?	1	$(2^{127}-5997)^2$	FaLoSá	CT-RSA 2014
Sandy	92000?	1	$(2^{127}-5997)^2$	FaLoSá	July 2014
Sandy	88916	2	$2^{127}-1$	**BeChLaSc**	Asiacrypt 2014
Haswell	161648	1	$2^{255}-19$	BeDuLaScYa	CHES 2011
Haswell	110740	2	$2^{127}-1$	BoCoHiLa	Eurocrypt 2013
Haswell	61712	1	2^{254}	OlLóArRo	CHES 2013
Haswell	60556	2	$2^{127}-1$	**BeChLaSc**	Asiacrypt 2014

pictures credit:

http://mitechnews.com/tag/personal-computer/

http://blogs.which.co.uk/technology/wp-content/uploads/2012/04/pc-versus-mac.jpg

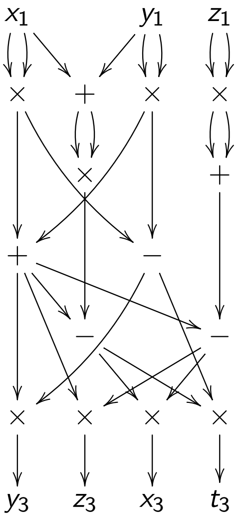
Based on

Curve41417: Karatsuba revisited
Joint work with Daniel J. Bernstein & Tanja Lange

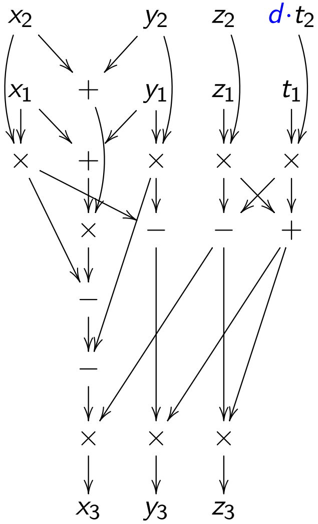
Curve41417 is a high-security elliptic curve
in Edwards form $x^2 + y^2 = 1 + 3617x^2y^2$
which is defined over prime field $\mathbf{F}_p$ where $p = 2^{414} - 17$.
Scalar multiplication is computed using signed fixed window
with width $\omega = 5$.

Point operations

Point doubling



Point addition



Polynomial multiplication

- Goal: Compute $P = AB$
  given $A = a_0 + a_1 t^n$ and $B = b_0 + b_1 t^n$
- Method 1: schoolbook
  $P = a_0 b_0 + (a_0 b_1 + a_1 b_0) t^n + a_1 b_1 t^{2n}$
- Method 2: Karatsuba ($8n-4$ additions)
  $P = a_0 b_0 + ((a_0 + a_1)(b_0 + b_1) - a_0 b_0 - a_1 b_1) t^n + a_1 b_1 t^{2n}$
- Method 3: refined Karatsuba ($7n-3$ additions)
  $P = (a_0 b_0 - a_1 b_1 t^n)(1 - t^n) + (a_0 + a_1)(b_0 + b_1) t^n$

Polynomial multiplication $\bmod Q$

- Goal: Compute $P = AB \bmod Q$
  given $A = a_0 + a_1 t^n$ and $B = b_0 + b_1 t^n$
- Method 1: schoolbook
  $P = a_0 b_0 + (a_0 b_1 + a_1 b_0) t^n + a_1 b_1 t^{2n} \bmod Q$
- Method 2: Karatsuba ($8n-4$ additions)
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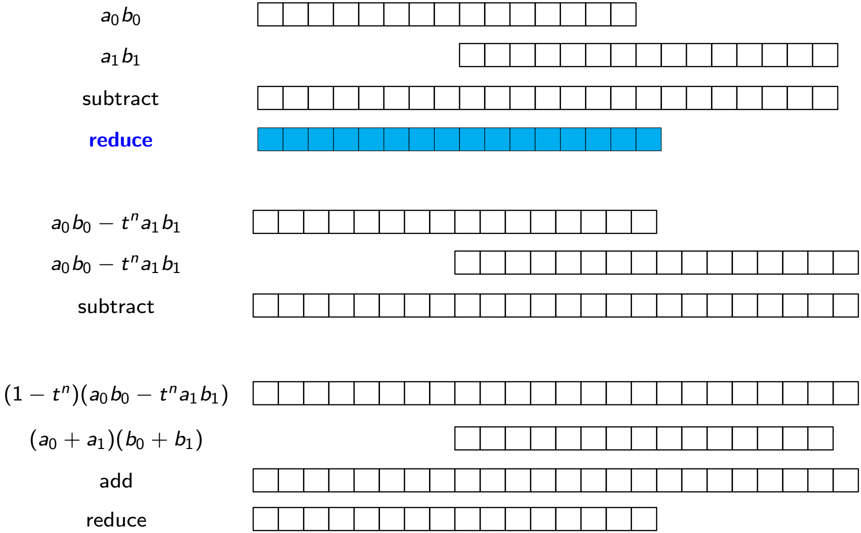
- Method 2: Karatsuba ($8n-4$ additions)
  $P = a_0 b_0 + ((a_0 + a_1)(b_0 + b_1) - a_0 b_0 - a_1 b_1) t^n + a_1 b_1 t^{2n} \bmod Q$
- Method 3: refined Karatsuba ($7n-3$ additions)
  $P = (a_0 b_0 - a_1 b_1 t^n)(1 - t^n) + (a_0 + a_1)(b_0 + b_1) t^n \bmod Q$
- Method 4: reduced refined Karatsuba ($6n-2$ additions) (new)
  $P = (a_0 b_0 - a_1 b_1 t^n \bmod Q)(1 - t^n) + (a_0 + a_1)(b_0 + b_1) t^n \bmod Q$



$a_0 b_0$
$a_1 b_1$
subtract
reduce
$a_0 b_0 - t^n a_1 b_1$
$a_0 b_0 - t^n a_1 b_1$
subtract
$(1 - t^n)(a_0 b_0 - t^n a_1 b_1)$
$(a_0 + a_1)(b_0 + b_1)$
add
reduce

Performance comparison

- OpenSSL (on Cortex-A8)

curve	# cycle on i.MX515	# cycle on Sitara
`secp160r1`	$\approx$ 2.1 million	$\approx$ 2.1 million
`nistp192`	$\approx$ 2.9 million	$\approx$ 2.8 million
`nistp224`	$\approx$ 4.0 million	$\approx$ 3.9 million
`nistp256`	$\approx$ 4.0 million	$\approx$ 3.9 million
`nistp384`	$\approx$ 13.3 million	$\approx$ 13.2 million
`nistp521`	$\approx$ 29.7 million	$\approx$ 29.7 million

- Curve41417 (security level above 2^{200})
  - $\approx$ 1.6 million cycles on i.MX515
  - $\approx$ 1.8 million cycles on TI Sitara

Making it run really fast

- Maximize usage of available vector multipliers
- Minimize cost from carries
  - use redundant representation
  - use non-integer radix, e.g., $2^{25.4}$ for Kummer on Cortex-A8
  - do not perform full carry
  - do parallel carry chain
- Eliminate redundancy inside field operations
  - precompute to reuse values
- Minimize overhead from permutations
  - organize data to fit instruction format
- Schedule instructions to keep CPU as busy as possible
- See papers for details
  - https://cr.yp.to/hecdh/kummer-20140218.pdf
  - https://cr.yp.to/ecdh/curve41417-20140706.pdf